Discounted Cash Flow (DCF) Analysis

Associated titles in the UBS Valuation Series:

- Evaluation methodology
- Cost of equity and of capital
- Dividend discount models
- Economic value added
- HOLT

Double-edged sword

Unlike traditional techniques, discounted cash flow (DCF) valuations take into account the explicit financial performance of all future years. However, such valuations are very sensitive to small changes in input data and while providing the scrupulous analyst with a very powerful tool, the unscrupulous can use the technique to justify just about any value they choose!

The advantage of a DCF valuation is that it allows the free cash flows that occur in all future years to be valued giving the 'true' or 'intrinsic' value of the business.

The disadvantage is that it requires accurate forecasts of future free cash flows and discount rates. Typically, explicit free cash flow forecasts are produced for 5 to 10 years. However, in general, the bulk (often 80% or more) of the value lies beyond this explicit forecast period, and this is captured in a terminal value calculation. The accuracy of this calculation is not only a function of the method used and the underlying assumptions, but also the level of cash flow at the end of the explicit forecast period.

Traditional valuation multiples are often used to calculate terminal value. However, it is difficult to estimate the fair value multiple for the company some 5 to 10 years hence, and this introduces an unacceptable level of inaccuracy.

A better approach to terminal value is to represent the free cash flows beyond the explicit forecast period in terms of a terminal growth rate and continue the DCF analysis in perpetuity. This growth must be consistent with the level of investment and return on investment, an issue which is often overlooked. An alternative is to treat the company as a cash cow and assume a decay rate in the gross cash flows, this eliminates the need to assess the ongoing level of investment.

With these points in mind, a DCF analysis can be used in three ways: (1) As a measure of absolute value. A DCF analysis will provide an absolute value and it is always useful to 'sense check' this against an implied traditional multiple (eg EV/EBITDA, PE, P/BV, etc). (2) As a tool in scenario analysis. These scenarios should reflect company specific factors rather than macro factors, which affect the value of all other companies in the market as well as the company being valued. Each scenario should use the same discount rate. (3) In a calculation of an implied discount rate based on current price and forecast free cash flows. This discount rate can be compared with those of other companies and/or a theoretical discount rate. This approach has the advantage that it permits relative valuations.

Overall, DCF valuations provide a more rigorous valuation approach than traditional techniques.

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Part 1

Practical guide to DCF valuation

Overview

Used by bankers and accountants, but rarely by analysts

Discounted cash flow (DCF) valuations are numerically intensive and, therefore, their use only became common-place when low-cost desktop computing was widely available in the 1980s. In addition, the technique was popularised by a number of business schools and tended, therefore, to be adopted by the academically accomplished but not necessarily the practically minded.

Most analysts have managed to avoid discounting techniques and there is still considerable resistance to its adoption as a standard valuation too. While this does not seem very enlightened, there are very sound reasons for their resistance.

Use discounting methods with care

Value is given by the cumulative present value of future cash flows...

Discounting techniques are based on the accepted economic principle that the value of an asset is represented by the cumulative present value of the future cash flows received by the asset holder. The cash flows are brought into present value terms by using a factor based on the discount rate.

Although discounting methods give, in economic terms, 'intrinsic' or 'true' value, the practical difficulties should not be overlooked.

...but future cash flows are difficult to assess...

The cash flows that will accrue to an owner may be the result of ordinary or special dividends, capital distributions, tax concessions, rights or entitlements, take-overs, etc. Some of these cash flows are not universal but are specific to certain owners. Moreover, the timing of these cash flows can be extremely uncertain.

...as is the discount rate

Similarly, the discount rate used to calculate the present value of the future 'investor' cash flows is an imponderable. The treatise on discount rates that line the walls of the utility regulators is testimony to this. The fact that small changes in discount rate can have significant value implications is often overlooked; for example; if the correct discount rate for a company with cash flows growing at 7% in perpetuity, was 10% but 9% was used in error the enterprise value would be a staggering 50% too high! Moreover, if debt represented say, half of the (correct) enterprise value, the equity value would be 100% too high!

What's more, all the variables are dynamic

Finally, the magnitude and timing of cash flows and the level of the discount rate are all dynamic. For example, each may be subject to sudden change as may occur as a result of the annual Budget, or to gradual changes as may occur with macro-economics change.

Good technique...but often impractical

Given the difficulty in forecasting cash flows and assessing the discount rate, the DCF technique is often impossible or impractical to use.

Generally, traditional techniques avoid these problems by implicitly assuming similar growth profiles and discount rates for a company and its comparators. There are, of course, circumstances where this assumption is quite wrong and this is illustrated in companion documents in the *UBS Valuation Series*.

Alternative approaches to DCF

DCF analysis opens the possibility of other approaches to valuation

Given the limitations of DCF valuation techniques, care should be taken in using them to assess absolute value. However, the technique opens the possibility of other approaches to valuation:

Implied market discount rate

Use current market value and forecast cash flows to calculate the implied market discount rate...

Given that a DCF relates three variables to one another; cash flows, discount rate and value, any two of these three are sufficient to determine the third. Thus, based on forecast cash flows and current market value, the implied discount rate can be calculated. This is an attractive process because of the three variables the assessment of the discount rate poses the greatest problem (current market value can be determined precisely and analysts would probably have a reasonable level of confidence in their cash flow forecasts).

An analysis on this basis would seek to rank companies according to implied discount rate and then either identify anomalies in the ranking in a subjective way or through an objective process of comparing implied discount rate to a theoretical rate.

Scenario analysis

...or examine the relative value of different scenarios

Probably the most powerful way of using a DCF analysis is to undertake scenario analyses. Typically, three cash flow scenarios would be forecast corresponding to key strategic alternatives; for example, for food retailers one might consider an aggressive price war, rational product pricing or some sort of collusion between the players. Each scenario would be valued using a DCF (the same discount rate would be used for each scenario). The value differences can then be assessed. The advantage of this technique is that:

- Compared to traditional valuation techniques that consider just a single point in time forecast, the effects of cash flow differences between scenarios that occur over several years can be valued.
- Any systematic errors in the cash flow forecast and/or discount rate will be neutralised when the percentage differences between the DCF valuation for each scenario is calculated.

It should be noted that the cash flow scenarios should model company specific (non-systematic) factors, while the systematic risks (interest rates, exchange rates, etc) are dealt with through the discount rate (refer to the *UBS Valuation Series: Cost of equity and capital*).

Practical use of DCF analysis for valuation

Much of this section is based on the theoretical work contained in *Part 2: Theoretical guide to DCF* of this document and reference should be made to this section if clarification of any item is needed.

How should DCF valuations be undertaken?

Value is given by the cumulative present value of future free cash flows

A DCF valuation is undertaken by calculating the cumulative present value of all future free cash flows. The present value is calculated using the discount rate. By free cash flows we mean the cash flows stated after capital expenditure and after tax but before net interest and dividends are paid. In other words, the cash flow available for debt and equity holders only. The value calculated in this way represents the enterprise value and, therefore, net debt needs to be removed to calculate the equity value. If, for whatever reason, the free cash flows being discounted are used to satisfy any other claims on the business then the total value of these claims should also be subtracted from the DCF value to calculate equity value. Great care should be taken under these circumstances to ensure the correct discount rate is used.

In general, DCF valuations give an absolute value and, therefore, the issue of comparators does not exist as it does with other techniques. However, a problem with using DCFs for calculating absolute values is that analysts' forecasts may be in error. In general, these errors tend to be systematic rather than random. Under these circumstances, DCFs are best suited to assessing the percentage differences between the values of various strategies as these differences will be unaffected by systematic errors. Finally, if the DCF technique is used to calculate an implied discount rate this can be compared with the implied discount rate for other stocks.

Where should DCF be used?

In theory, a DCF analysis is applicable to any company, however, certain practical difficulties exist. As a DCF valuation requires cash flows to be forecast over fairly long periods, the technique is difficult to apply if cash flows are highly cyclical, volatile or subject to dislocations. This said, other techniques also suffer from problems in these situations and a DCF approach is by no means any worse than other techniques and, in general, a lot better. One reason the technique is better is that the analyst is forced to explicitly forecast the cash flow profile and, therefore, address the business and strategic issues faced by the company.

DCF analysis is most suited to stable companies

The industries that most lend themselves to a DCF analysis are those with stable, forecastable cash flows and systematic risks that are easily defined.

What influences DCF valuations?

The valuation is influenced by the explicit cash flow forecasts...

...cost of capital...

A DCF valuation is influenced by the forecast cash flow profile and the cost of capital that is used to bring that profile into present value terms. The forecast cash flow profile is, obviously, influenced by the quality of the analyst's model and the quality of input assumptions. While the former is a mechanical quality, the latter is a cerebral quality. The second influence on value is the cost of capital and this too requires a degree of mechanical calculation and cerebral input.

...and terminal value

In practice, cash flows cannot be forecast for all time and, therefore, some assumptions are made as to the cash flow characteristics beyond the explicit forecast period. These assumptions are used to calculate a terminal value and clearly these too will influence the calculation of value.

Most DCF valuations break down because cash flow forecasts are poor, terminal value calculations are inappropriate and discount rates are not estimated properly.

Advantages and disadvantages of DCF valuation

The main advantages of a DCF analysis is that it allows different cash flow profiles to be explicitly valued. Offsetting this very significant advantage is the effect that small errors in the forecast cash flow profile and discount rate will have on calculated value.

A summary of the main advantages and disadvantages of a DCF valuation are given in Table 1.

Table 1: Advantages and disadvantages of DCF valuation

Advantages	Disadvantages
● Theoretically correct	● Accurate forecasts are required (this includes forecasts of capital spend on assets, investments or acquisitions)
● Forces the analyst to be rigorous in modelling future cash flows	● Care needs to be taken in identifying claims on the business other than debt and equity, such as pensions
● Explicitly takes account of the value impact of future profile of cash flows	● Requires accurate estimate of the (after-tax) cost of capital
	● Little consensus between users of the technique on the method of calculating, or the level of, the cost of capital
	● Easily abused by unscrupulous users
	● Often seen as inaccessible by anyone other than the valuer
	● Often over 80% of the value lies beyond the explicit forecast period and this value is calculated using terminal value techniques, these techniques are at best approximate and at worst totally inappropriate.

UBS' approach to DCF

In this section, we describe our approach to a DCF analysis. First, we consider the proper definition of cash flow, followed by the calculation of the discount rate and then the actual process of discounting. Next, we discuss alternative ways of calculating terminal value, and finally, we calculate the value of equity.

Defining cash flow

Cash flows should be stated...

...after capital expenditure and tax...

...but before interest and dividends

A discounted cash flow must always use cash flow stated after capital expenditure but before interest and dividends (ie the free cash flow to the firm or FCFF). Occasionally, cash flows are taken after maintenance capital expenditure, this is incorrect and should, therefore, never be done. Cash flow must always be stated after full capital expenditure and should include the effects of matured debt, conversion of loan notes, exercise of share options etc. Some analysts take cash flows before tax in which case a pre-tax cost of capital should be used. If cash flows are stated after-tax (but still before interest and dividend) then an after-tax cost of capital should be used, this is the standard UBS approach. The tax charge to be used for the after-tax cash flow should take the actual cash tax paid and add to this the tax shield available to the company as a result of interest paid. We prefer to use the after tax cost of capital as its components are measurable in the capital market. The cash flow is illustrated in Table 2.

Table 2: Calculating cash flow

	Components of cash flow
	UBS operating profit
Add back	Depreciation
Add back	Other non-cash items (amortisation, change in provisions)
(Less)/add back	(Increase)/decrease in operating working capital
Less	Capital expenditure
Less	Net acquisitions/disposals*
Add back	Shares issued less shares re-purchased
Less (add back)	Currency variations (if positive)*
Less (add back)	Other income (if positive)*
⇨	Pre-tax cash flow
Less	UBS tax
Less	Tax shield resulting from interest payments**
Add back	Reclaimable withholding tax***
⇨	Free cash flow to the firm**** (sometimes called after tax cash flow)
Less	Net interest paid (cash net interest as opposed to P&L net interest)
Less	Dividends paid (cash dividends as opposed to P&L dividends)
Add back	Tax shield resulting from interest payments**
Less	Reclaimable withholding tax***
=	Change in net debt/cash

* These items are often neglected, however, the cash flows must be based on a thorough forecast for the company and should include the effect of matured debt, conversion of convertible loan notes, exercise of share options etc.
** This is not part of the corporate cash flow but must be included for consistency with the calculation of WACC *(see UBS valuation series: cost of equity and capital).*
*** This is not part of the corporate cash flow but is included for consistency with the calculation of the COE (COE is a gross return, see *UBS valuation series: cost of equity and capital*).
****Preferred cash flow for discounting purposes.

If cash flows are stated before any item other than net interest and dividends then the capitalised value of this item together with the capitalised value of net interest (ie net debt) should be subtracted from the DCF value to determine the equity value. For example if an analyst believed a company faced a windfall profit tax at some point in the future but was uncertain as to the timing of the cash out flow, the tax could be ignored providing the present value of the tax was subtracted from the DCF value. Of course, if timing is unknown it is difficult to accurately calculate present value, however, a guesstimate is usually satisfactory.

There is an important point here; if the cash outflow is not model, then the model may add value to (or subtract value from) the 'phantom' retained cash. Clearly, in the example above, not only should the present value of the tax be subtracted from the DCF value but also the present value of the value added to (or taken from) the 'phantom' retained cash. While we have considered a windfall profit tax there are many similar one-off or multi-year charges that are intentionally omitted from the cash flow forecast. Where this happens, DCF value needs to be adjusted.

Calculating discount rate

The discount rate is the weighted average cost of equity and debt

The cost of capital represents the returns demanded by all the capital providers (eg debt, convertible debt, preference shares, ordinary shares, etc). Not surprisingly, the cost of capital is a weighted average of all of these different costs of finance and is known as the weighted average cost of capital (WACC). The weighting is based on the proportion (in market value terms) of the total capital represented by each instrument. For example, if the company was financed by just debt and equity then:

$$WACC = \frac{G}{(1+G)}.COD.(1-T) + \frac{1}{(1+G)}.COE$$

In this equation, COD is the cost of debt and COE is the after-corporation-tax pre-investor tax cost of equity and must be adjusted to take account of equity risk introduced through the financial structure of the company. G is the gearing of the company (net debt/equity) based on market values of debt and of equity and T is the marginal corporate tax rate. Note that if net debt is negative (ie net cash) the same formula can be used providing the cash flow is stated before net interest received and the net cash is added to the DCF value to obtain equity value.

Sometimes, analysts use the accounting value of debt and equity when calculating WACC. This is not correct - always use market values.

Cost of debt

The cost of debt is based on the risk free rate and a risk premium...

The cost of debt is a straight-forward calculation:

COD = R_f + debt risk premium.

The risk-free rate (R_f) is given in Table 3. The debt risk premium is relatively small, at between 40bp and 120bp. For large, high-quality businesses with high interest cover a debt risk premium at the lower end of this range is appropriate and vice versa. To simplify the calculation, we use one of three risk premia:

Table 3: Estimating the debt risk premium

Debt risk premium	Comment
0.4%	High-quality corporate, high interest cover, typically AA+ rated
0.8%	Average-quality corporate, average interest cover, typically A rated
1.2%	Low-quality corporate, low interest cover, typically BBB rated

Cost of equity

...so is the cost of equity - although the risk premium is quite different

The cost of equity is the total return demanded by equity investors on their investment. Investors will demand higher returns on their investments as the risk to those returns increases and vice versa. This risk/reward relationship is a fundamental concept used in calculating the cost of equity. A common way of expressing this relationship is via a model known as the capital asset pricing model (CAPM). This model, which is discussed in more detail in the *UBS Valuation Series: Cost of equity and of capital*, breaks down the return into two principal components:

- The returns on 'risk free' investments (government debt). This is generally known as the risk free rate (R_f).
- The additional return required to compensate for the uncertainty associated with investing in the company.

This latter component is further broken down into two parts:

- The return that the market as a whole will deliver over and above the risk free rate. This is known as the market risk premium (R_p).
- The risk of the company relative to that of the market. This is known as the company's beta (β). This risk represents both the sensitivity of the underlying business (systematic business risk) to macro influences and the risk to equity investors from the capital structure of the company (financial risk).

The way these components are combined is straightforward and is shown in the equation below:

$$COE = R_F + \beta \,.\, R_P$$

Risk-free rate is taken as the 10-year maturity government bond

The risk-free rate is based on the 10-year maturity government bond. There are many arguments to support longer-dated bonds. However, provided the same bond is used to determine the risk-free rate and as part of the market premium calculation, the differences in the final value for COE are relatively immaterial. It is simply convenient to use a 10-year bond because they are available in all European markets.

Market risk premium

The market risk premium is the difference between the expected market return and the risk-free rate

We take the risk premium as a forward-looking premium based on the expected long-run returns of the market (a detailed description of the calculation of the market risk premium is given in the *UBS Valuation Series: Cost of equity and of capital* less the risk-free rate (see previous section). Typically, this varies between 2% and 4% depending on the market.

Each quarter, UBS will publish the risk-free rate and the forward-looking risk premium for the key European markets in its regular strategy document entitled '*European Equity Market Indicators*', which is also available on our web page: **www.ubs.com/research**.

Beta (β)

The risk factor is calculated by the analyst based on...

The final component of the COE calculation is the risk measure (beta or β). We estimate, within certain guidelines, a forward looking β. To introduce some consistency into the calculation of β across various sectors and countries, we base the risk measurement on two factors:

...systematic business risks, and...

Systematic business risk

Generally, business risk is the risk faced by the entire company ignoring the risks that may result from any financing decision. This risk is split into two components; risks that are specific to the company and risks that affect all companies. Examples of the former would be the risk of a competitor changing pricing policy or the threat of industrial action while examples of the latter would be changes in inflation or in interest rates. It is only the risks that affect all companies that are taken into account when estimating β. These risks are known as systematic risks. These risks should be estimated relative to an 'average' company. Risks specific to a company should be addressed through a scenario analysis, ie via financial modelling, not through changes to the discount rate (for more information refer to *UBS Valuation Series: Cost of equity and of capital*).

...financial risks...

Financial risk

This is the equity risk that is introduced as a result of the capital structure (gearing) of the company. A formula for the effects of gearing on β is given in *UBS Valuation Series: Cost of equity and of capital*. The level of gearing should be estimated relative to the market average.

Table 4 is constructed by considering categories of business and financial risk. The maximum and minimum βs from Table 4 are 1.6 and 0.4, respectively which is a slightly narrower range than the range of historical βs. This is appropriate as βs tend to be mean reverting (ie move towards 1.0 over time).

Table 4: Company risk premium: β (β= 1 + A + B, see below)

Systematic business risk		Financial risk	
Risk category of company rel. to market avg.	Risk adjustment (A)*	Company gearing %**	Risk adjustment (B)***
Lower quintile	-0.4	-20	-0.2
Next quintile	-0.2	0	-0.1
Next quintile	+0.0	20	+0.0
Next quintile	+0.2	40	+0.1
Upper quintile	+0.4	60	+0.2

* The systematic business risks relative to the 'average' company in the market.
** Based on market values of debt and equity.
*** The financial risk adjustment assumes a gearing level (based on market values) of 20% for the market. If economic gearing differs from this then for each 20 percentage points of company gearing above/below the market the financial risk adjustment can be taken as +/-0.1.

In using Table 4, if there is a significant change in gearing over the forecast period then the analyst should adjust the COE for these changes.

...cost of other types of capital need to be considered

Cost of other types of capital

While we have considered equity and debt finance, other types of finance can be used eg; preference shares, convertibles, etc. These types of finance will share some of the characteristics of equity and of debt and it is best to cost them on this basis. A simple alternative, where the other instruments are not a significant part (ie less than 10%) of the capital base, is to assign each type of instrument to either debt or equity.

Calculating discounted value

Value equals present value of future 'explicit' free cash flows and the terminal value

A cash flow forecast of, preferably, five years or more should be used and this needs to be brought into present value terms by using the discounting technique. Value that occurs beyond this explicit forecast period is approximated by a terminal value calculation and this too needs to be discounted to present value terms.

This discounting process is shown below:

$$V = \frac{FCFF_1}{(1+W)} + \frac{FCFF_2}{(1+W)^2} + \frac{FCFF_3}{(1+W)^3} + \frac{FCFF_4}{(1+W)^4} + \frac{FCFF_5}{(1+W)^5} + \frac{TV_6}{(1+W)^5}$$

In calculating discounted value, the discounting process above (FCFF) represents the free cash flow to the firm (ie cash flow after capex and after tax but before interest and dividend) and the subscript denotes the year in which it occurs, W denotes the weighted cost of capital and TV denotes the terminal value. In the example given above, the terminal value is the value in year five based on the cash flow (or earnings, or dividend, etc) in year six and beyond and this is why the discount rate is raised to the power of five. The valuation given above pre-supposes that the first cash flow forecast occurs 12 months from the date of the valuation. If this is not the case then a mid-year adjustment needs to be applied to the valuation, to reflect the incidence of the first cash flow earlier than 12 months hence (this adjustment is discussed in *Part 2: Effects of errors in cash flow and discount rate*).

Terminal value

Terminal value sometimes, but not always, needs to be fully diluted

The cash flows must be based on a thorough forecast for the company and should include the effect of matured debt, conversion of convertible loan notes, exercise of share options etc. To the extent that these do not occur during the explicit forecast periods the terminal value should be calculated on a diluted basis. We use one of two approaches to terminal value, more approaches are discussed in *Part 2: Calculating terminal value.*

Constant cash flow growth

Terminal value is often based on the assumption that the long run free cash flow growth is constant...

By considering the growth in free cash flow (cash flow after capex and after tax but before interest and dividend) beyond the end of the explicit forecast period T to be constant, we can calculate the terminal value. This is shown in the Table 5.

Table 5: Ratio of terminal value to free cash flow (%)*

Discount rate (%)	long term growth in free cash flow to the firm (%) 0	2	4	6	8	10
6	16.7	25.5	52.0	**	**	**
8	12.5	17.0	26.0	53.0	**	**
10	10.0	12.8	17.3	26.5	54.0	**
12	8.3	10.2	13.0	17.7	27.0	55.0
14	7.1	8.5	10.4	13.3	18.0	27.5

* The terminal value still needs to be brought into present value terms by dividing by $(1 + WACC)^T$
** Value is infinite if long term growth is equal to or greater than the discount rate!

...long run growth can be based on growth in the underlying market...

An important variable in the terminal value calculation is the long run growth in free cash flow. Small errors in the estimation of this growth rate can have a huge input on the terminal value, particularly for high-growth companies. As an example, the terminal enterprise value for a long-term growth of 8% and discount rate of

10% would be 50% more than that of the same company but assuming a long-term growth rate of 7%. Moreover, if net debt amounted to half of the enterprise value of the 7% growth scenarios, the equity value of growth was assured to be 8% would be twice that of the 7% case.

Unfortunately, long term growth rates are often difficult to forecast, and so to is the free cash flow. However, if we assume that after the explicit forecast period the company is in a steady state (ie return on equity and return on capital are each constant), the growth of the company can only be driven by the growth in net new investment (capital expenditure over and above maintenance capital expenditure). Thus, under steady state conditions the growth rate will be dictated by the rate of net new investment which will be the same as, or close to, the growth rate of the company's market (ie growth rate in new car sales or telephone calls or planes ordered, etc). Unfortunately, the assumed capital expenditure in the final year of the explicit forecast is often inconsistent with the long term growth rate. In *Part 2: Calculating terminal value*, we present a method (equation 11) that can be used to calculate the capital expenditure that would be associated with the long run growth.

...but the capital expenditure must be consistent with this growth

The formula that underlies Table 5 is given in *Part 2*.

Zero value added

An alternative to constant cash flow growth is 'zero value added'...

...this method assumes that no value is added to investments beyond the explicit forecast period

Terminal value based on 'zero value added' assumes that at the end of the explicit forecast period, management earn an economic return on future investments that is identical to the cost of capital. In other words, if DM40m of the gross cash flow (cash flow before capex, dividend and interest but after tax) is invested in new assets the present value of the future cash flows from these assets will be DM40m. On this basis, new investments do not add value and we can, therefore, simply discount the gross cash flows from existing assets. For convenience, we assume that these cash flows decline linearly over the remaining life of the asset base. The ratio of the terminal value to **gross** cash flow is given in Table 6.

Table 6: Ratio of terminal value to gross cash flow*

Discount rate (%)	Remaining asset life (years) 5	10	15	20	25	30
6	2.2	4.0	5.5	6.8	7.8	8.7
8	2.1	'3.7	5.0	6.1	6.9	7.6
10	2.0	3.5	4.6	5.5	6.1	6.6
12	1.9	3.3	4.3	5.0	5.5	5.9
14	1.9	3.1	4.0	4.5	5.0	5.3

* The terminal value still needs to be brought into present value terms by dividing by $(1 + WACC)^T$, where 'T' is the length (in years) of the explicit forecast period.

This approach neither requires a growth forecast nor a capital expenditure forecast...

A further problem with the constant growth model is the level of fuel cash flow at the end of the explicit forecast period. In particular, the capital expenditure (the difference between gross and free cash flow) is often not consistent with the assumed long-term growth rate.

... for these reasons, we prefer this approach

The 'zero value added' approach to terminal value is our preferred approach as the gross cash flow is easier to calculate than the free cash flow (capital expenditure does not need to be calculated) and the remaining life of the asset base is usually very easy to estimate. Moreover, the variation in terminal value to gross cash flow ratio varies much less than terminal value to free cash flow. However, this whole approach does presuppose that management neither add nor destroy value beyond the explicit forecast period.

The formula that underlies Table 6 is given in *Part 2*.

Calculating the value of equity

To calculate equity value, all non equity value needs to be subtracted from the DCF value

The DCF analysis, if undertaken properly, will give the value of the entire company, in other words the enterprise value. This value is split between debt, convertibles, preference shares and ordinary shares, and any other method of capital provision. Thus, to calculate the value of the equity (ordinary shares) it is necessary to subtract from the DCF value, the value of all non-equity. The value of non-equity should be based on the market value not book value, this is discussed in *Part 2: Calculating the value of equity*.

Once the equity value is calculated then this should be divided by the shares outstanding (do not dilute the shares as the conversion of share options and convertible loan notes should have been taken into account in the cash flows or terminal value used to calculate the DCF) to calculate a target share price.

Part 2

Theoretical guide

Effects of errors

While a DCF analysis will give the intrinsic value of the forecast cash flows at a particular discount rate, neither of these two quantities is easily calculated. Moreover, cash flows are not forecast for all time and, therefore, at the end of the explicit forecast period, a terminal value is calculated which often relies on traditional valuation techniques. Any one or all of these three items:

- Cash flow forecast.
- Terminal value.
- Discount rate.

can be in error.

The effects of errors on value are addressed below. For convenience, the first two of these items are treated together.

Cash flow forecast and terminal value

Errors in terminal value are common and very significant...

...even if the method of calculating terminal value is correct...

...it is still impacted by errors in the cash flow forecast

Tables 7 and 8 illustrate one of the potential problems with DCF analyses; little value is general associated with the explicit forecast period. For typical discount rates (after-tax, weighted-average cost of capital of 8% to 12%), free cash flow growth rates (5% to 7%) and explicit forecasts of five years, less than 20% of the value of the company will result from the cash flows in the explicit forecast period. Even for an explicit forecast period of 10 years, only about 30% of value is generated by the explicit cash flows. For high growth companies, the figures could be as low as 5% or 10% for discount rates of 8% and 12%, respectively. Clearly, for most practical purposes a DCF only skims the surface of valuation and the bulk of the value lies in the terminal value. Indeed, except for low-growth companies, it would probably be far better to put effort into calculating terminal value than trying to forecast five or 10 years of cash flow!

Table 7: Percentage of value accumulated in explicit forecast period*

Duration of explicit forecast (years)	Growth rate / discount rate* 0.0	0.3	0.6	0.9
1	7	5	3	1
2	14	10	6	1
3	21	15	9	2
4	26	19	11	3
5	32	23	14	4
6	37	27	17	4
7	42	31	19	5
8	46	35	21	6
9	50	38	24	6
10	54	41	26	7

* Using a discount rate of 8%.

Table 8: Percentage of value accumulated in explicit forecast period*

Duration of explicit forecast	Growth rate / discount rate 0	0.3	0.6	0.9
1	11	8	4	1
2	20	14	8	2
3	29	21	12	3
4	36	27	16	4
5	43	32	20	5
6	49	37	23	6
7	55	42	26	7
8	60	46	30	8
9	64	50	33	9
10	68	54	35	10

* Using a discount rate of 12%.

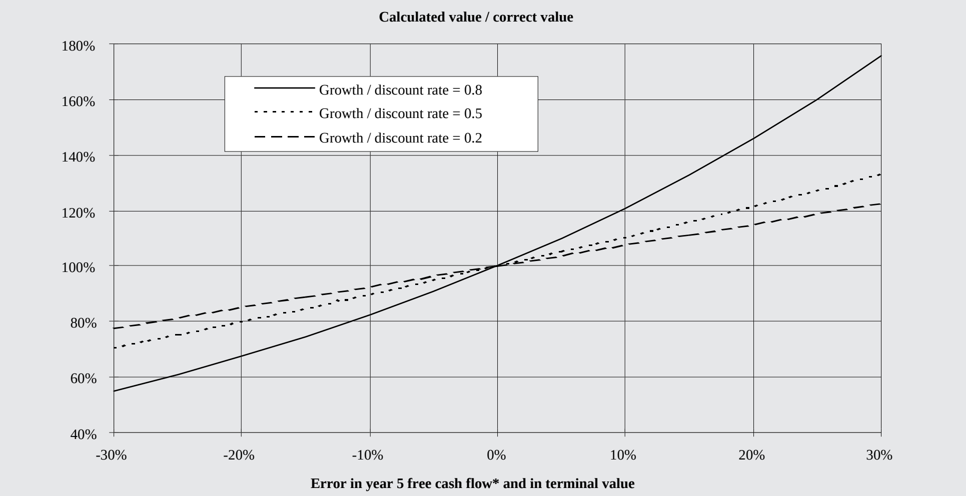
Effect of forecast errors

Often analysts' cash flow forecasts are in error

A quick skim through our archive of research (UBS and other brokers) suggests that to within 80% confidence limits, analysts forecast cash flows one year hence to an accuracy of ±30%. We have been generous in Chart 1 and assume that analysts can forecast the fifth year of cash flows to within an accuracy of±30% and therefore also terminal value. The resulting errors in valuation could quite easily range from 50% to 180% for a typical growth rate and discount rate.

While these errors are large, by any measure, they are errors in the overall value of a company. To value the equity, the analyst needs to subtract the non-equity components. In general, these are fairly easily assessed with a reasonable degree of accuracy. Thus, any errors in the overall value of the company will be amplified when considering just the equity alone. For example, if the 50% to 180% value errors noted above occurred in a company with 50% debt (by market value) the error in the equity value would be 0% to 230% off the mark! To put this into context, it would be as if an analyst proposed a fair value P/E of 0.0 or 37.0x for the German market (actual P/E of about 16x). The analyst would, quite rightly, be laughed out of court.

Chart 1: Same errors in the fit year forecast and terminal value



Calculating terminal value

The terminal enterprise value represents the value of the cash flows that accrue after the explicit forecast period. More often than not, most of the value of a business is actually represented by the terminal value. This was illustrated in Tables 10 and 11 in the previous section.

There are several ways in which terminal value can be calculated:

- Constant cash flow growth.
- Zero value added.
- Constant (operating) profit growth.
- Market multiples and financial ratios.

Our preferred approach is 'zero value added'

Our preferred approach to calculating terminal value is through 'zero value added' although constant cash flow growth is a close second followed by constant operating profit growth. Market multiples and financial ratios are too vague.

Each of these is considered in the following sections.

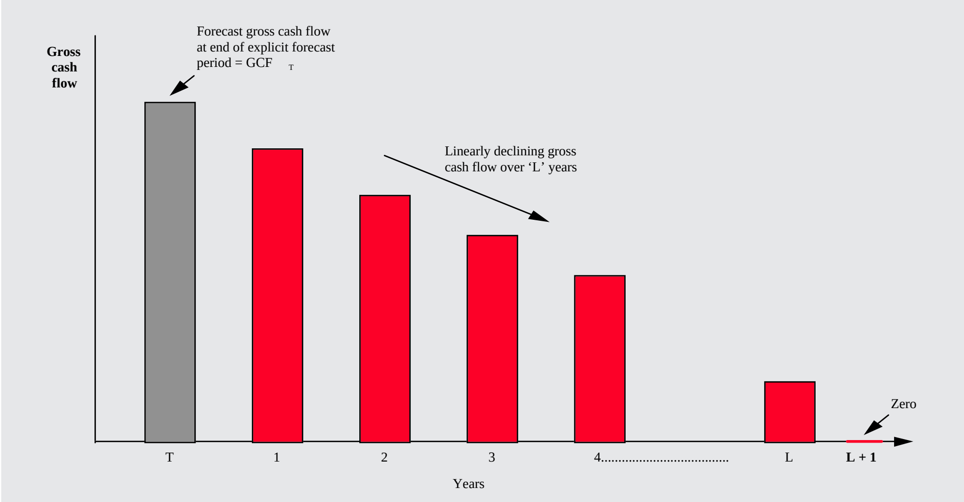
Zero value added

'Zero value added' assumes no value is added to investments after the explicit forecast period

Terminal value based on 'zero value added' assumes that at the end of the explicit forecast period, management earn an economic return on future investments that is identical to the cost of capital. In other words, if DM80m of the gross cash flow is invested in new assets, the present value of the future cash flow from these assets will be DM80m. On this basis, new investments do not add value and we can, therefore, simply discount the gross cash flows from existing assets. For convenience we assume that these cash flows decline linearly to zero when the last productive asset has reached the end of its life (this would not be the average remaining life across all the assets but would be the longest remaining life of the productive assets). This is illustrated in Chart 2.

It is quite possible that some value might remain when all productive assets reach the end of their lives. For example, the value of land and buildings. This value (appropriately discounted) should be added to the linearly declining gross cash flows. However, for simplicity we assume, in the calculations below that the residual value is zero.

Chart 2: Declining cash flows underlies 'zero value added'



From Chart 2 we can see that the value (TV) of the declining gross cash flow from GCFFT at the end of the explicit forecast period to zero at L + 1 is given by:

$$TV = \sum_{n=1}^{n=L} \frac{GCFF_T\,(1 - n/(L+1))}{(1+W)^n} \qquad (1)$$

Where n is the year (first year after the end of the explicit forecast period n = 1, second year n = 2, etc), W is the weighted average cost of capital and L is the life remaining at the end of the explicit forecast period of the longest life asset.

This equation can be re-written as:

$$TV = \frac{GCFF_T}{L+1} \sum_{n=1}^{n=L} \frac{L+1-n}{(1+W)^n}$$

or

$$TV = \frac{GCFF_T}{L+1}\left((L+1)\sum_{n=1}^{n=L}\frac{1}{(1+W)^n} - \sum_{n=1}^{n=L}\frac{n}{(1+W)^n}\right)$$

This equation can be reduced to:

$$TV = \frac{GCFF_T}{W^2(1+W)^L(1+L)}\left((1+W)^L.(WL-1)+1\right) \qquad (2)$$

In present value terms, the terminal value is given by:

$$\frac{TV}{(1+W)^T} = \frac{GCFF_T.((1+W^L).(WL-1)+1}{W^2(1+L).(1+W)^{(L+T)}} \qquad (3)$$

Where T is the duration of the explicit forecast period in years and GCF is this gross cash flow at the end of the explicit forecast period, W is the weighted average cost of capital and L is the remaining life of the asset base.

Table 5 presented in *part 1* gave the ratio of the terminal value to gross cash flow of the firm. This table was based on a slight modification to equation (3); the terminal value is not brought into present value terms and is divided by GCFF, ie:

$$\frac{TV}{GCFF_T} = \frac{((1 + W^L).(WL - 1) + 1)}{W^2 (1 + L).(1 + W)^L} \tag{4}$$

Constant cash flow growth

The most important part of the constant free cash flow growth model is...

By considering the growth g in free cash flow to the firm (FCFF) beyond the end of the explicit forecast period T to be constant, we can calculate the terminal value. This is shown below:

$$\frac{TV}{(1 + WACC)^T} = \frac{FCFF\,(1 + g)}{(WACC - g)\,(1 + WACC)^n}$$

In this equation WACC represents the weighted average cost of capital and the FCFF is the last forecast for the explicit forecast period.

An important variable in the terminal value calculation is the long-run growth in free cash flow. If we assume that after the explicit forecast period the company is in a steady state (ie return on equity and return on capital are each constant) the growth of the company can only be driven by the growth in net new investment (capital expenditure over and above maintenance capital expenditure). Thus, under steady state conditions the growth rate will be dictated by the rate of net new investment which will be the same as, or close to, the growth rate of the company's market (ie growth rate in new car sales or telephone calls or planes ordered, etc).

While we can determine growth in free cash flow to the firm by considering the 'macro' environment in which the company operates, it is helpful to ensure that the level of free cash flow to which the growth is applied is consistent with the rate of investment. This can be done by ensuring that the ratio of free cash flow to gross cash flow (cash flow after capital expenditure to cash flow before capital expenditure) is consistent with the assumed growth rate. This ratio is calculated below.

We can relate the gross cash flow (GCFF) to the gross book value (GBV) of assets through the cash flow return on gross book value (gross cash return (GCR)), ie:

$$GCR = \frac{GCFF_1}{GBV_0} \tag{5}$$

While the gross cash flow return may not be a familiar concept, it is the same as the initial EBITDA over capital employed (CE) for a project. However, for a project EBITDA/CE will rise as the capital employed is eroded by accumulated depreciation. Thus, the advantage of a gross cash flow return is that it provides a return figure that is unaffected by the rate of investment and is, therefore, constant (in the absence of changing inflation).

In addition to equation (1) we can say that the gross book value one year to the next is related through the capital expenditure (CX) i.e:

$$GBV_1 = GBV_0 + CX_1 \tag{6}$$

From equation (2) we can write the growth in gross book value (which will be the same as the growth in gross cash flow if gross cash flow return is constant) in terms of capital expenditure.

$$\frac{GBV_1 - GBV_0}{GBV_0} = g = \frac{CX_1}{GBV_0} \tag{7}$$

Equation (5) can be substituted into equation (7) to give growth in terms of return and gross cash flow:

$$g = \frac{CX_1 . GCR}{GCFF_1} \tag{8}$$

Finally, we know that the free cash flow to the firm (FCFF) is related to the gross cash flow as follows:

$$FCFF_1 = GCFF_1 - CX_1 \tag{9}$$

therefore, substituting (9) in (8) gives:

$$g = \left(\frac{GCFF_1 - FCFF_1}{GCFF_1} \right) GCR$$

or in more general terms:

$$g = \left(1 - \frac{FCFF}{GCFF} \right)$$

It may be more convenient to express the ratio of free cash flow to gross cash flow in terms of growth and gross cash return:

$$FCFF / GCFF = 1 - g / GCR \tag{10}$$

or to express the level of capital expenditure in terms of gross cash flow, growth and cash return:

... the level of capital expenditure

$$CX = \frac{g . GCFF}{GCR} \tag{11}$$

Equations (10) or (11) should be used in conjunction with the constant cash flow growth equation to ensure the free cash flow is consistent with the forecast gross cash flow and assumed growth.

Constant operating profit growth

Cash flows can be expressed in terms of the operating profit and net new investment (in fixed assets and operating working capital), this is illustrated in Table 9 which gives a simplified cash flow statement.

Table 9: Simplified cash flow statement

	Cash flow item		
+	After tax operating profit (NOPAT)*		
+	Depreciation	}	
-	Capital expenditure	}	Net increase in assets** (=ΔA)
-	Increase in operating working capital	}	
=	After-tax free cash flow to the firm		
-	Tax shield on interest payments		
-	Cash interest		
-	Cash dividend		
=	Change in net debt		

* The tax charge is taken as P&L tax plus the interest tax shield.
** Net increase in asset is the same as net new investment or change in capital employed.

Thus, we can write:

$$FCFF_n = NOPAT_n - \Delta A_n \qquad (12)$$

However, $NOPAT_n$ can be expressed as a return on accounting capital employed, ie:

$$NOPAT_n = ROCE_n \,.\, BV_{n-1} \qquad (13)$$

Where $ROCE_n$ represents the return on capital employed and BV_{n-1} represents the opening capital employed.

If we assume that ROCE remains constant overtime, which would be the case in steady-state conditions, then:

$$NOPAT_n - NOPAT_{n-1} = ROCE \,.\, (BV_{n-1} - BV_{n-2}) \qquad (14)$$

or

$$NOPAT_n = NOPAT_{n-1} + ROCE \,.\, \Delta A_{n-1} \qquad (15)$$

In addition, with constant ROCE the growth in NOPAT (g_{op}) will be constant. This growth can be expressed as:

$$g_{op} = (NOPAT_n - NOPAT_{n-1}) / (NOPAT_{n-1}) \qquad (16)$$

By substituting equation (16) in equation (15) the net increase in assets can be expressed in terms of operating profit, growth and return;

$$\Delta A_{n-1} = g_{op} \,.\, (NOPAT_{n-1}) / ROCE \qquad (17)$$

Equation (17) can now be substituted into equation (14) to obtain the free cash flow in terms of operating profit, growth and return;

$$FCFFn = NOPAT_n - g_{op} \,.\, NOPAT_n / ROCE$$

or;

$$FCFF_1 = NOPAT_1 \,(1 - g_{op} / ROCE) \qquad (18)$$

If equation (18) is substituted in the constant cash flow growth model, equation (13), and bearing in mind that in steady state conditions $g_{CF} = g_{op} = g$, we can write the value of the entire business under conditions of constant growth as;

$$\text{Fair enterprise value} = \frac{\text{NOPAT . (ROCE - g)}}{\text{ROCE . (WACC - g)}} \qquad (19)$$

Where the net operating profit after tax (NOPAT) is for the first forecast year.

Market multiples and financial ratios

Market multiples are not recommended

Market multiples are sometimes used to calculate a terminal enterprise value. The most common multiple is EV/EBITDA although sometimes EV/sales is used. While this approach is appealing because it is quick and simple, it has shortcomings. For example, the level of the multiple that is chosen is often very subjective, moreover, given the importance of the terminal value to total value, one might as well value the entire company on the basis of the market multiple. Consequently, we do not recommend the use of market multiples for calculating terminal values in a DCF analysis.

Market multiples for terminal value of equity

Sometimes market multiples are used to asses the terminal value of equity rather than the value of the enterprise. If these multiples are used it is necessary to subtract from the enterprise value over the explicit forecast period, the present value of the non equity over the same period to give the value of the equity over the explicit forecast period. This can then be added to the terminal value of equity to give total equity value. This is shown in Table 10 Below:

Table 10: Assessing terminal value of equity

	Component of value
+	Present value of free cash flow over the explicit forecast period
-	Present value of non-equity cash flow over the explicit forecast period
=	Present value of the equity over explicit forecast period
+	Terminal value of equity based on market multiples
=	Total value of equity

In general, one of the following four multiples will be used:

- P/E (price to earnings per share).
- Yield.
- P/cash flow.
- P/NAV (price to net asset value).

While a market multiple approach to terminal value is appealing as analysts feel they have a better grasp of these than EV multiples, it really should be regarded as a poor approach and, if at all possible, other approaches should be used. Our reasoning is as follows; a terminal multiple is difficult to assess. It will bear no relation to the current multiple, which reflects the cash flow both through and beyond the explicit forecast period. Rather, the terminal multiple reflects the cash flows beyond the explicit forecast period only. In this respect, the choice of terminal multiple will be at least as, if not more, judgmental then using a current multiple to value the share and ignoring the discounted cash flow approach altogether. This, is further reinforced if the reader bears in mind that 80% or more of the total value of a share is represented by the terminal value and if this is based on a spurious market multiple then there can be little confidence in the total value.

In preference to a terminal market multiple we would prefer to use the constant cash flow growth model presented on the previous page.

An alternative to using market multiples is the use of embedded value measures. At UBS two measures have been used:

- Liquidation value.
- Replacement cost.

As both of these methods focus on the value of tangible assets they are inappropriate where there is considerable intangible assets such as people (service industries) or brands (some consumer products based industries).

Liquidation value

Only use liquidation value if the assets will be liquidated

Liquidation value is based on the estimated proceeds from the sale of the assets of the business less the liabilities at the end of the explicit forecast period. This value is not at all easy to calculate and is, in any event, usually highly subjective. It is often best used in industries where the assets are in a tradable form; typically financial assets (insurance policies for example), commodities (for example a tin mine with definable reserves) or, to a lesser extent, land and property. The approach is not particularly suited to businesses comprising large physical asset bases such as oil refineries, electricity generators, shipping fleets, etc, where the liquidation value may be nothing more than the scrap value. Finally, this approach should really only be used if the assets are likely to be liquidated.

Replacement cost

Replacement cost is good but very difficult to calculate

Replacement cost can be interpreted in two ways, firstly; the cost of replacing the existing assets with like assets; secondly; replacing the existing assets with assets which will provide the end customer with same product or service (economists often refer to this method as the optimum net deprival value). An example of this latter method would be the replacement of a fixed telecoms network with a mobile system or the replacement of a sub optimal electricity distribution network (for example; one that was built to distribute to industrial customers that are no longer in existence) with an optimal one. In both approaches, the ‘new’ cost should be depreciated to reflect the remaining economic (as opposed to accounting) life of the assets. The replacement cost approach is particularly suited to companies that have large physical asset bases such as chemicals companies, telecoms companies, electricity companies etc.

Calculating the value of equity

Non-equity claims on the business

A DCF analysis gives the value of the entire business...

A DCF valuation gives the value of the entire business. To calculate the value of the equity, the value of non-equity claims on the business need to subtracted from the value of the business.

...to get equity value subtract...

There are two main non-equity claims on the business:

- Debt and any other non-equity funding such as preference shares
- Unfunded liabilities such as German pensions.

Debt

...debt...

There are two common ways of valuing the net debt of a business:

- Book value of net debt.
- Market value of net debt.

In the UK and continental Europe most analysts use the book value while in the US most analysts use market value. There is little justification for using the book value of debt, indeed no more than for using the book value of equity! A straight forward way of examining this is to consider two identical companies in every respect except that one company is funded by debt that carries a lower coupon than the other. Clearly, the pre interest cash flows will be identical and, therefore, the value of the businesses will be identical. However, in the case of the company that has lower interest costs there will be more cash available for shareholders and the value of its equity would be higher. Therefore, despite having the same book value of debt each has a different market value of debt. The relationship between market value and book value is shown below.

Market value of debt = book value of debt × coupon / current interest rate

It is important to note that 'current interest rate' in the above formula means the current rate that would prevail for debt with a term structure that was identical to the remaining term structure of the book debt.

In Europe, the difference between book value and market value has often been overlooked because debt was usually issued at floating rate. With floating rate debt, the ratio of coupon to current interest rates is unity. In the US, it is more common for debt to be issued at fixed rate and, therefore, market value can differ from book value. However, it is increasingly common for European companies to issue fixed rate debt.

Other non-equity funding

...other non-equity funding...

If the cash flows that have been discounted have been stated before any funding (excluding debt) such as;

- Preference shares
- Off-balance sheet debt (or more generally, non-equity)
- Warrants, etc.

Then the capitalised value (cumulative present value of future costs) of these costs should be subtracted from the DCF value.

In addition to these examples there is a grey area that might include leasing, derivative contracts etc. It is crucial that the analyst really understands what cost of funding has been included and what has been excluded in the free cash flow, and adjust the enterprise value accordingly.

Unfunded liabilities

...unfunded liabilities

Any liability that would be funded out of the cash flows that are used in the DCF valuation must be removed in order to calculate the value of equity. One such liability is the pension liability of German companies. German companies pay pensions on a cash basis out of a pension provision that is funded through a P&L charge. However, as the cash charge (the charge impacting cash flows) meets only the demands of the existing pensioners a liability is created for additional future pensioners. Of course, future cash flow forecasts could include future cash pension charges and the growth in these could be included in the terminal value calculation. However, rather than forecast annual cash pension over the explicit forecast periods, it is better to state cash flows before pension costs and subtract the total present value of the pension liability from the calculated enterprise value. Unfortunately, where German companies have issued financial statements under US GAAP it is clear that, in general, the balance sheet pension provision is insufficient to meet the future pension liabilities and so the analyst will still need to estimate total pension liability.

Part 3

Appendices

Appendix A: Terminal value

The what and why of terminal value

In all discounting methods of valuation it is necessary to forecast the quantity being valued (EVA, dividend, free cash flows, etc) for all time. Quite clearly, forecasting over an infinite time period is impractical. In fact, if such an analysis was undertaken eventually the forecast would settle down to steady-state conditions or trend. Fortunately, we can simplify the discounting process if steady-state conditions have been achieved. This simplified discounting process provides a terminal value. **Thus, a terminal value is calculated once steady-state conditions have been achieved and such conditions will signal the end of the explicit forecast period.**

Calculating terminal value in general

The general form that a discounting approach to valuation takes is:

$$V = \sum_{n=1}^{n=\infty} \frac{Z_n}{(1 + DR)^n} \qquad \text{(A:1)}$$

Where Z is the quantity being discounted (it could be EVA, dividend or free cash flow or a number of other measures), n denotes the year and DR is the discount rate (usually either the weighted average cost of capital or the cost of equity or whatever is consistent with the quantity being discounted).

When steady state conditions have been achieved, rates of return will be constant as will growth rates.

With constant returns, growth rates will be driven by the growth in invested capital or in equity. It is common practice to take these growth rates as the long-run growth in the stock market unless there is a clearly justifiable reason for adopting a different growth rate. In the next section on terminal value for EVA calculations we show how to calculate the terminal growth rate from the conditions at the end of the explicit forecast period. If the growth so calculated is different from that of the market or from expectations steady-state conditions may not have been reached.

Under conditions of constant growth:

$Z_n = Z_0 \times (1 + g_Z)^n$

Where g_Z is the year on year growth rate of Z. Thus equation (A:1) becomes:

$$V = \sum_{n=1}^{n=\infty} \frac{Z_0 \times (1 + g_Z)^n}{(1 + DR)^n} \qquad \text{(A:2)}$$

If we now expand equation (A:2) we obtain:

$$V = \frac{Z_0 \times (1 + g_Z)^1}{(1 + DR)^1} + \frac{Z_0 \times (1 + g_Z)^2}{(1 + DR)^2} + \frac{Z_0 \times (1 + g_Z)^3}{(1 + DR)^3} + \ldots \qquad \text{(A:3)}$$

If we now multiply each side of equation (A:3) by $(1 + g_Z)/(1 + DR)$ we obtain;

$$V \times \frac{(1 + g_Z)}{(1 + DR)} = \frac{Z_0 \times (1 + g_Z)^2}{(1 + DR)^2} + \frac{Z_0 \times (1 + g_Z)^3}{(1 + DR)^3} + \ldots \qquad \text{(A:4)}$$

The next step is to subtract equation (A:4) from equation (A:3) to give;

$$V - V \times \frac{(1 + g_Z)}{(1 + DR)} = \frac{Z_0 \times (1 + g_Z)^1}{(1 + DR)^1} \qquad \text{(A:5)}$$

Equation (34) can be re-written as:

$$\frac{V \times (1 + DR) - V \times (1 + g_Z)}{(1 + DR)} = \frac{Z_0 \times (1 + g_Z)}{(1 + DR)}$$

or, simplifying further:

$$V = \frac{Z_0 \times (1 + g_Z)}{(DR - g_Z)} = \frac{Z_1}{(DR - g_Z)} \qquad \text{(A:6)}$$

Equation (A:6) is the general formula used for terminal value purposes. Remember that when using this formula, Z_0 refers to the last value of Z in the explicit forecast period and V is the value of all future values of Z beyond the explicit forecast period. Consequently, V needs to be converted to a present value by multiplying by the present value factor for the last forecast year (=1/(1+DR)a) where 'a' typically lies between five and 10 and represents the explicit forecast period ie between five and 10 years.

A special case of equation (A:6) is for $g_Z = 0$, which gives:

$$V = \frac{Z_0}{DR} = \frac{Z_1}{DR} \qquad \text{(A:7)}$$

This is known as the value of an annuity.

Appendix B: Present value calculations

Calculating present value

If an investor had the choice between £15k now or £20k, guaranteed, in four years time which should be chosen! Hopefully, the investor would calculate the amount that would be returned in four years time if the £15k was invested in a totally safe (guaranteed) scheme. For example, a government bond might pay 7% interest and if the interest was reinvested the £15k would become;

Table 11: Over time, capital increases in value

Year	Value at start of year (£k)	Increase in value	Value at end of year (£k)
1	15.00	15.00 × 1.07	16.05
2	16.05	16.05 × 1.07	17.17
3	17.17	17.17 × 1.07	18.38
4	18.38	18.38 × 1.07	19.66

On the basis of the calculation above, the investor should opt for the £20k in four years time.

Another way of interpreting Table 11 is to say that a guaranteed payment of £19.66k in four year time is equivalent, to £15k now, ie:

£15k = £19.66k / (1 + 7%)^4 = £19.66k / 1.311

Where '^4' means raised to the power of 4 (or multiplied by itself 4 times ie (1 + 7%) × (1 + 7%) × (1 + 7%) × (1 + 7%)).

The process of bringing the future cash payment into an equivalent money of today (or present value) is known as **discounting**. The **discount rate** is 7% and the **present value factor** is 0.763 (=1 / 1.311)

The example given above presupposes that that the investor would invest the £15k in something that would give the same level of security that was offered by the £20k payment in four years time. What if the investor had the opportunity to invest now in a speculative venture that might, in four years time, have a realisable value of £65k? While the investor has potential to make a lot more money there is no guarantee that the potential will be realised. The most sensible thing to do is to consider the return that would be available from some alternative investment opportunity of similar risk. Assume that this analysis was done and that, typically, similar investment opportunities returned 35% pa. The investor would complete the analysis as follows:

Discount rate	35%
Present value factor	1 / ((1 + 35%)^4) = 0.301
Present value of £65k	£19.57k

Clearly, the investor should not invest in the speculative venture as it is described above.

In this example the **discount rate** is 35% and this is a **risk adjusted** discount rate that represents the **opportunity cost of capital**.

Discounting a stream of income

The previous example considered a single payment at some time in the future. Discounting a stream of income simply treats the incidence of each payment as a single payment and adds the present value of these together. This is illustrated in Table 12.

Table 12: Discounting a stream of income

Year	Cash payment	Formula for present value factor	Discount rate (DR)	Present value factor	Present value	Cumulative present value
1	15.0	1/(1+DR)^1	11%	0.901	13.5	13.5
2	27.0	1/(1+DR)^2	11%	0.812	21.9	35.4
3	42.0	1/(1+DR)^3	12%	0.731	30.7	66.1
4	12.0	1/(1+DR)^4	12%	0.636	7.6	73.8
5	9.0	1/(1+DR)^5	12%	0.567	5.1	78.9

Discounting cash flows over uneven time periods

So far we have assumed that the first cash payment is one year away from today and that every other payment follows at internals of one year. We need to consider two alternative possibilities.

- First payment is less than a year away but payments thereafter are at intervals of one year.
- The intervals between payments are all different.

First payment less than a year away

If an annual discount rate has been determined, which is usually the case, we first need to reduce it into a past-year discount rate. This is straight forward; if the annual discount rate is DR then the monthly discount rate 'dr' would be related to it as follows:

$$\frac{1}{(1 + \mathrm{dr})^{12}} = \frac{1}{1 + \mathrm{DR}}$$

or $(1 + \mathrm{dr}) = (1 + \mathrm{DR})1/12$

In other words, the present value factor for a single payment after one year based on annual discount rates must be equivalent to the present value factor for a single payment in 12 months time based on monthly discount rates.

If we now had cash payments of 'a', 'b', 'c'... occurring after 3, 15, 27, ...months respectively the cumulative present value (cum pv) would be given by:

$$\text{Cum pv} = \frac{a}{(1 + \mathrm{DR})^{3/12}} + \frac{b}{(1 + \mathrm{DR})^{15/12}} + \frac{c}{(1 + \mathrm{DR})^{27/12}} + \ldots$$

or

$$\text{Cum pv} = \frac{1}{(1 + \mathrm{DR})^{-9/12}} \left(\frac{a}{(1 + \mathrm{DR})^{1}} + \frac{b}{(1 + \mathrm{DR})^{2}} + \ldots\right)$$

Thus, if a series of annual cash flows begin in 'm' months time then we can calculate the cumulative present value as if the first payment occurred in 12 months time and multiply this cumulative present value by (1 + DR) ^((12 - m) / 12). We term this adjustment as **'starting adjustment to present value'.**

Mathematical note: When one number 'raised to the power of ...' is multiplied by the same number raised to a different power of ... the two 'powers' are added eg:

$4^2 \times 4^3 = (4 \times 4) \times (4 \times 4 \times 4) = (4 \times 4 \times 4 \times 4 \times 4) = 4^5$

When one number 'raised to the power of ...' is divided by the same number raised to a different power the two 'powers' are subtracted eg:

$4^2 / 4^3 = (4 \times 4) / (4 \times 4 \times 4) = 1/4 = 4^{-1}$

Different intervals between payments

In effect, the previous section dealt with this issue. If, for example we had 3 payments a, b and d occurring 4, 7 and 35 months from now, the cumulative present value would be

$$\text{Cum pv} = \frac{a}{(1 + DR)^{4/12}} + \frac{b}{(1 + DR)^{7/12}} + \frac{c}{(1 + DR)^{35/12}}$$

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